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Bianchi Type-II Bulk Viscous String Cosmological Models in General Relativity

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Abstract Some Bianchi type II bulk viscous string cosmological models with electromagnetic field are investigated. To get a determinate solution, we assume the shear (σ) is proportional to the expansion (θ), which leads to a supplementary condition $B = lA^n$, between metric potentials, is used where A and B are function of time alone. A particular solution for $n = 0$ is also discussed. The physical and geometrical implications of the models are discussed.

Keywords Cosmic strings · Viscous fluid · Bianchi type II models

1 Introduction

Bianchi type II models play an important role in current modern cosmology, for simplification and description of the large scale behavior of the actual universe. Kibble [4] and Zel'dovich [13] have investigated that strings are one of the sources of density perturbation which are required for the formation of large scale structure of the universe. These strings possess stress energy and are coupled to the gravitational field. Letelier [5] and Stachel [11] have initiated the general relativistic treatment of the strings. Consequently, many relativists have obtained exact solutions which described homogeneous string cosmological models with different Bianchi symmetries. Asseo and Sol [1] emphasized the importance of Bianchi type II universe. Bali and Anjali [2] has investigated Bianchi type I magnetized string cosmological model in general relativity. Rao et al. [8] studied Exact Bianchi Type II, VIII and IX string cosmological models in Saez-Ballester Theory of Gravitation. Singh and Agrawal [10] studied Bianchi type II, VIII and IX models in scalar tensor theory under the

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assumption of a relationship between the cosmological constant (Λ) and scalar field (ψ). Most cosmological models assume that the matter in the universe can be described by 'dust' (a pressure-less distribution) or at best a perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of expanding universe [14, 15]. Bali and Pradhan [16] have investigated Bianchi type-III string cosmological models with bulk viscous fluid for massive strings. To obtain the determinate model of the universe, they assumed that the coefficient of bulk viscosity ξ is inversely proportional to the expansion θ in the model and expansion θ in the model is proportional to the shear σ.

Singh [9] also studied string cosmology with electromagnetic fields in Bianchi type II, VIII and IX space times. Bali and Dave [3] have investigated Bianchi type IX string cosmological models with bulk viscosity. Pradhan et al. [6] have studied LRS cosmological models of Bianchi type II representing clouds of geometrical as well as massive strings. Pradhan et al. [7] also studied string cosmological models for bulk viscous fluid. Recently, Tyagi et al. [12] have investigated Bianchi type II string cosmological model assuming the condition $\rho = k\lambda$, where ρ is the energy density and λ is the string tension density and k is constant.

Motivated by the situations discussed above, in this paper, we have investigated some Bianchi type II bulk viscous string cosmological model in general relativity. To get determinate solution, we have used a supplementary condition $B = lA^n$ between the metric potentials. A particular solution for $n = 0$ is also discussed. The physical and geometrical implications of the models are discussed.

2 The Metric and Field Equations

We consider Bianchi type II space-time in the form

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy - xdz)^2 \tag{1}$$

where A and B are function of t-alone.

In a co-moving coordinate system, we have

$$v^i = (0, 0, 0, 1), \qquad x^i = \left(\frac{1}{A}, 0, 0, 0\right) \tag{2}$$

The energy-momentum tensor (T_i^j) for cloud of string with bulk viscous fluid is given by as

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi v^\ell_{;\ell}(g_i^j + v_i v^j) \tag{3}$$

here ξ is the bulk coefficient of viscosity, $\rho = \rho_\rho + \lambda$, the proper energy density for cloud of string with particle attached to them. ρ_ρ, particle energy density, λ, is the string tension density, v^i is the four velocity of particles and x^i is the unit space like vector representing the direction of string satisfying

$$v_i v^i = -1 = -x_i x^i \tag{4}$$

The Einstein's field equations for a system of strings are given by Letelier [5]

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \tag{5}$$

for the line element (1) leads to following system of equations

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{B^2}{4A^4}=\lambda+\xi\theta \tag{6}$$

$$\frac{2\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}-\frac{3}{4}\frac{B^2}{A^4}=\xi\theta \tag{7}$$

$$\frac{\dot{A}}{A^2}+\frac{2\dot{A}\dot{B}}{AB}-\frac{B^2}{4A^4}=\rho \tag{8}$$

3 Solution of Field Equations

Here A, B, λ, ρ and ξ are five unknowns, we need two extra conditions.

We have assumed two conditions:

(i)

$$\xi\theta = M \quad \text{(constant)} \tag{9}$$

i.e. the coefficient of bulk viscosity is inversely proportional to the expansion (θ).

(ii) Shear (σ) is proportional to expansion (θ). This leads to

$$B=\ell A^n \tag{10}$$

relation between metric potentials A and B, ℓ and n are constants.

Using (9) and (10) in (8), we get

$$\frac{2\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}=M+\frac{3}{4}\ell^2A^{2n-4} \tag{11}$$

This leads to

$$2A\ddot{A}+\dot{A}^2=MA^2+\frac{3}{4}\ell^2A^{2n-2} \tag{12}$$

Let us assume

$$\dot{A}=\eta \tag{13}$$

$$\ddot{A}=\eta\frac{d\eta}{dA} \tag{14}$$

$$\frac{d}{dA}\{\eta^2A\}=MA^2+\frac{3}{4}\ell^2A^{2n-2} \tag{15}$$

After integration (15) leads to

$$\eta^2=\frac{MA^2}{3}+\frac{3}{4}\frac{\ell^2A^{2n-2}}{(2n-1)}+\frac{L}{A} \tag{16}$$

where L, is the constant of integration.

Equation (16) leads to

$$dt=\left[\frac{MA^2}{3}+\frac{3}{4}\frac{\ell^2A^{2n-2}}{(2n-1)}+\frac{L}{A}\right]^{-1/2}dA \tag{17}$$

Hence using proper transformation of coordinates metric (1) reduces to

$$ds^2 = -\left[\frac{MT^2}{3} + \frac{3}{4}\frac{\ell^2 T^{2n-2}}{(2n-1)} + \frac{L}{T}\right]^{-1} dT^2 + T^2(dX^2 + dZ^2) + \ell^2 T^{2n}(dY - XdZ)^2 \quad (18)$$

4 Some Physical and Geometrical Features

The energy density (ρ), the string tension density (λ), the expansion (θ), coefficient of bulk viscosity (ξ), the shear (σ), the particle density (ρ_p) for the model (18) in the presence bulk viscosity are given by.

$$\rho = (2n+1)\left(\frac{M}{3} + \frac{L}{T^3}\right) + \frac{\ell^2(n+1)T^{2n-4}}{(2n-1)} \quad (19)$$

$$\lambda = \frac{M}{3}(n^2+n-2) + \frac{L}{2T^3}(2n^2-n-1) + \frac{\ell^2(3n^2+n-2)T^{2n-4}}{2(2n-1)} \quad (20)$$

$$\theta = (n+2)\left[\frac{M}{3} + \frac{L}{T^3} + \frac{3}{4}\frac{\ell^2 T^{2n-4}}{(2n-1)}\right]^{1/2} \quad (21)$$

$$\xi = \frac{M}{(n+2)}\left[\frac{M}{3} + \frac{L}{T^3} + \frac{3}{4}\frac{\ell^2 T^{2n-4}}{(2n-1)}\right]^{-1/2} \quad (22)$$

$$\sigma = \frac{(1-n)}{\sqrt{3}}\left[\frac{M}{3} + \frac{L}{T^3} + \frac{3}{4}\frac{\ell^2 T^{2n-4}}{(2n-1)}\right]^{1/2} \quad (23)$$

$$\rho_p = \frac{M}{3}(3+n-n^2) + \frac{L}{2T^3}(3+5n-2n^2) + \frac{\ell^2(4+3n-n^2)T^{2n-4}}{2(2n-1)} \quad (24)$$

In the absence of bulk viscosity i.e. when $M \to 0$ then line element (1) reduce to

$$ds^2 = -\left[\frac{3}{4}\frac{\ell^2 T^{2n-2}}{(2n-1)} + \frac{L}{T}\right]^{-1} dT^2 + T^2(dX^2 + dZ^2) + \ell^2 T^{2n}(dY - XdZ)^2 \quad (25)$$

The energy density (ρ), the string tension density (λ), the expansion (θ), coefficient of bulk viscosity (ξ), the shear (σ), the particle density (ρ_p) for the model (25) are given by

$$\rho = \frac{(2n+1)L}{T^3} + \frac{\ell^2(n+1)T^{2n-4}}{(2n-1)} \quad (26)$$

$$\lambda = \frac{\ell^2(3n^2+n-2)T^{2n-4}}{2(2n-1)} + \frac{(2n^2-n-1)L}{2T^3} \quad (27)$$

$$\theta = (n+2)\left[\frac{L}{T^3} + \frac{3}{4}\frac{\ell^2 T^{2n-4}}{(2n-1)}\right]^{1/2} \quad (28)$$

$$\rho_p = \frac{L}{2T^3}(3+5n-2n^2) + \frac{\ell^2(4+n-3n^2)T^{2n-4}}{2(2n-1)} \quad (29)$$

$$\sigma = \frac{(1-n)}{\sqrt{3}}\left[\frac{L}{T^3} + \frac{3}{4}\frac{\ell^2 T^{2n-4}}{(2n-1)}\right]^{1/2} \quad (30)$$

5 Discussion

The energy condition $\rho \geq 0$ in pressure of bulk viscous fluid leads to

$$(2n+1)\left[\frac{M}{3}+\frac{L}{T^3}\right]+\frac{\ell^2(n+1)T^{2n-4}}{(2n-1)} \geq 0 \tag{31}$$

When $T \to \infty$, $\rho = \frac{(2n+1)M}{3}$ and $\theta = \frac{(n+2)M}{3}$ when $n < 2$, due to presence of bulk viscous fluid. Since $\mathrm{Lim}_{T\to\infty}\frac{\sigma}{\theta} \neq 0$ when $n < 2$, hence model does not isotropize for large values of T.

As $T \to 0$ then $\rho \to \infty$, $\theta \to \infty$, the model starts with a big bang at $T = 0$. The energy condition $\rho \geq 0$ for the model (25) leads to

$$\frac{(1-4n^2)L}{\ell^2(n+1)} \geq T^{(1-2n)} \tag{32}$$

Thus the model (25) exists during the time span given by (32). The model starts big bang at $T = 0$ and the expansion in the model decreases as time increases. $\mathrm{Lim}_{T\to\infty}\frac{\sigma}{\theta} \neq 0$, hence model does not isotropize for large values of T.

6 Special Case

When $n = 0$, equation (16), leads to

$$\eta^2 = \frac{MA^2}{3} - \frac{3}{4}\frac{\ell^2}{A^2} + \frac{L}{A} \tag{33}$$

To get a realistic situation and the nature of model, we assume $L = 0$. Now (33) becomes

$$\eta^2 = \frac{MA^2}{3} - \frac{3}{4}\frac{\ell^2}{A^2} \tag{34}$$

This leads to

$$\sqrt{\frac{3}{M}}\frac{AdA}{\sqrt{A^4 - \frac{9\ell^2}{4M}}} = dt \tag{35}$$

On integrating this leads to

$$A^2 = \frac{3\ell}{2\sqrt{M}}\cosh\left(2\sqrt{\frac{M}{3}}t + N\right) \tag{36}$$

where N is constant of integration.

After suitable transformation of coordinates the metric (1) reduces to the form

$$ds^2 = -\frac{3}{4M}dT^2 + \frac{3\ell}{2\sqrt{M}}\cosh T(dX^2 + dZ^2) + \ell^2(dY - XdZ)^2 \tag{37}$$

6.1 Some Physical and Geometrical Features

The energy density (ρ), the string tension density (λ), the expansion (θ), the shear (σ), the particle density (ρ_p) for the model (37) are given by

$$\rho = \frac{\tanh^2 T}{4} - \frac{M \operatorname{sech}^2 T}{9} \tag{38}$$

$$\lambda = \frac{4M + 9(1 + \cosh^2 T) - 36M \cosh^2 T}{36 \cosh^2 T} \tag{39}$$

$$\theta = \tanh T \tag{40}$$

$$\sigma = \frac{\tanh T}{2\sqrt{3}} \tag{41}$$

$$\rho_p = \frac{36M \cosh^2 T - 8M - 18}{36 \cosh^2 T} \tag{42}$$

The reality condition $\rho > 0$ leads to

$$\frac{\tanh^2 T}{4} - \frac{M \operatorname{sech}^2 T}{9} > 0 \tag{43}$$

This leads to

$$\sinh T > \sqrt{\frac{4M}{9}} \tag{44}$$

Therefore for the realistic model, we take span of time given by (44).

When $T \to \frac{\pi}{2}$, $\rho \to \infty$, $\lambda \to \infty$.

The expansion in the model increases as the time increases.

$\operatorname{Lim}_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, hence model does not isotropize for large values of T.

The models (17) and (24) have Point Type singularity when $n > 0$. (MacCallum [17]).

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